

# PLASMON NEUTRINOS EMISSION IN A STRONG MAGNETIC FIELD

## II. Longitudinal Plasmons

V. CANUTO, C. CHIUDERI\* and C. K. CHOU

*Institute for Space Studies, NASA Goddard Space Flight Center, New York, U.S.A.*

(Received 13 January, in final form 24 July, 1970)

**Abstract.** The decay of a longitudinal plasmon into two neutrinos is studied in the presence of a strong magnetic field. Contrary to the transverse case, for longitudinal plasmons the existence of a new mode, entirely dependent on the magnetic field, greatly enhances the energy loss at high densities. Denoting by  $Q_H$  and  $Q_O$  the neutrino energy losses with and without magnetic field respectively, the situation is as follows: at  $H \simeq 10^{11}$  G and  $T \geq 10^9$  K,  $Q_O \gg 10^5 Q_H$  for  $\rho < 10^{11}$  g cm $^{-3}$ , while  $Q_H \gg 10^{10} Q_O$  for  $\rho > 10^{11}$  g cm $^{-3}$ . A second physically interesting feature is the anisotropic character of the energy loss which is highly peaked along the field lines, giving rise to a shorter cooling time in that direction than in any other.

### 1. Introduction

In a previous paper (Canuto *et al.*, 1970) (hereafter referred to as Paper I) the effect of an intense magnetic field on the decay rate of a transverse plasmon into a neutrino-antineutrino pair was considered. It was shown that the effect is usually negligible in the regions of astrophysical interest. This is mainly due to the fact that the magnetic field enters into the formulas only through the cyclotron frequency  $\omega_c = (eH/mc) = (H/H_q)(mc^2/\hbar)$  which is very small in comparison with the plasma frequency,  $\omega_p$ ,  $(\omega_c/\omega_p) \simeq 25(H/H_q)\rho_6^{-1/2}$ .

In this paper we will study the analogous problem of the decay of a longitudinal plasmon into a neutrino-antineutrino pair in the presence of a strong magnetic field. As we shall show, there is a new longitudinal plasma mode, which reduces to  $\omega = \omega_c$  for  $\theta = 0$ , greatly enhancing the energy loss due to neutrinos emission. Since the technique of the computation closely follows that of Paper I, we will give here only an outline of the general procedure, referring to Paper I for more details.

### 2. The Longitudinal Plasma Modes

The longitudinal plasmon vector potential  $A_\mu^l$  can be written as

$$A_\mu^l(\mathbf{k}) = \sum_{\mathbf{k}} N_\gamma^{1/2} \{a(\mathbf{k}) e^{ikx} + a^\dagger(\mathbf{k}) e^{-ikx}\} e_\mu^l(\mathbf{k}), \quad (1)$$

where

$$N_\gamma = \frac{4\pi\hbar c^2}{\Omega\omega^2} \left| \frac{\text{Tr} \lambda_{ij}^l}{\frac{\partial A^l}{\partial \omega}} \right|; \quad (2)$$

\* On leave of absence from the University of Florence (Italy).

$\lambda_{ij}^l$  and  $\Lambda^l$  are the cofactors, and the determinant of the modified Maxwell operator  $\Lambda_{ij}^l$  given by Melrose (1968) as

$$\left. \begin{aligned} \Lambda_{ij}^l &= \Lambda^l K_i K_j; \quad K_i \equiv \frac{k_i}{|\mathbf{k}|}; \\ \Lambda^l &= K_i \Lambda_{ij} K_j = K_i \varepsilon_{ij} K_j; \end{aligned} \right\} \quad (3)$$

and  $\Lambda_{ij}$  is the Maxwell operator discussed in Paper I: namely,

$$\Lambda_{ij} = \left( \frac{ck}{\omega} \right)^2 [K_i K_j - \delta_{ij}] + \varepsilon_{ij}; \quad (4)$$

$\varepsilon_{ij}$  being the dielectric tensor of the medium. As discussed in Paper I we shall choose  $\varepsilon_{ij}$ , in the form given by Stix (1962) as\*

$$\left. \begin{aligned} \varepsilon_{ij} &= \begin{vmatrix} S & -iD & 0 \\ iD & S & 0 \\ 0 & 0 & P \end{vmatrix}, \\ S &= \frac{1}{2}(R + L), \quad D = \frac{1}{2}(R - L), \\ R &= 1 - \frac{\omega_p^2}{\omega^2} \left( \frac{\omega}{\omega - \omega_c} \right), \\ L &= 1 - \frac{\omega_p^2}{\omega^2} \left( \frac{\omega}{\omega + \omega_c} \right), \\ P &= 1 - \frac{\omega_p^2}{\omega^2}, \\ \omega_p^2 &= \frac{4\pi N e^2}{m}, \quad \omega_c = \frac{eH}{mc}; \end{aligned} \right\} \quad (5)$$

$N$  being the electron number density. The ion component has been neglected.

The dispersion relation can be written formally as

$$\Lambda^l = A = S \sin^2 \theta + P \cos^2 \theta \quad (6)$$

or

$$\tan^2 \theta = - \left( \frac{P}{S} \right). \quad (7)$$

Inserting the explicit expressions of  $P$  and  $S$  in Equation (7) we have

$$\tan^2 \theta = - \frac{(\omega^2 - \omega_c^2)(\omega^2 - \omega_p^2)}{\omega^2(\omega^2 - \omega_p^2 - \omega_c^2)}. \quad (8)$$

The possible longitudinal modes are then given by ( $\omega_h^2 = \omega_p^2 + \omega_c^2$ ) as

$$\omega_{\pm}^2 = \frac{1}{2} \{ \omega_h^2 \pm [\omega_h^4 - 4\omega_p^2 \omega_c^2 \cos^2 \theta]^{1/2} \},$$

\* It can be shown that Equation (5) can be obtained quantum mechanically for zero temperature and in the long wave length approximation (Canuto and Ventura, 1970).

which at  $\theta=0$  and  $\theta=\pi/2$  becomes

$$\begin{aligned}\theta = 0: \quad \omega_-^2 &= \omega_c^2, \quad \omega_+^2 = \omega_p^2; \\ \theta = \frac{\pi}{2}: \quad \omega_-^2 &= 0, \quad \omega_+^2 = \omega_h^2 = \omega_p^2 + \omega_c^2;\end{aligned}$$

since  $\omega_h^2 \simeq \omega_p^2$ , the  $\omega_+$  mode is practically independent of  $H$  and isotropic, and is therefore the one studied by Adams *et al.* (1963). All the quantities referring to this mode will be hereafter denoted by a subscript zero.

On the other hand the mode  $\omega_-$  at  $\theta=0$  depends solely on  $H$  and this is precisely the mode which dominates for high densities. Analogously, the presence of this mode will be characterized by a subscript  $H$ .

### 3. Computation of the Energy Loss

The computation of the energy loss proceeds much in the same way as for the transverse case, with only slight modifications. We shall not elaborate on the details but will indicate the results appropriate to the longitudinal case. The energy loss is given by the formula

$$Q = \sum_{\text{spin}} \frac{\Omega^2}{(2\pi\hbar)^6} \iint d^3\mathbf{p}_1 d^3\mathbf{p}_2 \frac{|S|^2}{\Omega T} (E_1 + E_2), \quad (9)$$

where  $\mathbf{p}_1$ ,  $E_1$  and  $\mathbf{p}_2$ ,  $E_2$  are the neutrino and antineutrino final momentum and energy. The relevant  $S$ -matrix for the neutrino-antineutrino decay of a longitudinal plasmon is:

$$\begin{aligned}S = \frac{g}{e\sqrt{2}} \frac{N_\gamma^{1/2}}{\hbar c} (2\pi)^4 \delta^4 \left( \frac{p_1 + p_2}{\hbar} - k \right) [\Pi_{\alpha\beta}(\mathbf{k}, \omega) e_\alpha] \\ \times [\bar{u}(p_2) \gamma_\beta (1 + \gamma_5) v(p_1)], \quad (10)\end{aligned}$$

where all the symbols have the same meaning as in Paper I. The polarization tensor  $\Pi_{\alpha\beta}(\mathbf{k}, \omega)$  is defined as

$$\Pi_{\alpha\beta}(\mathbf{k}, \omega) = \frac{e^2}{\hbar c} \frac{1}{(2\pi\hbar)^4} \int d^4p \text{Tr} \{ \gamma_\alpha G(p) \gamma_\beta G(p+k) \}, \quad (11)$$

and is related to the dielectric tensor  $\varepsilon_{\alpha\beta}$  through the relations (Tsytovich, 1961)

$$\begin{aligned}\Pi_{ij} &= \frac{i}{4\pi} \left( \frac{\omega}{c} \right)^2 [\varepsilon_{ij} - \delta_{ij}], \\ \Pi_{i4} &= i \left( \frac{\omega}{c} \right)^{-1} \Pi_{ij} k_j, \\ \Pi_{44} &= - \left( \frac{\omega}{c} \right)^{-2} \Pi_{ij} k_i k_j.\end{aligned} \quad (12)$$

Squaring  $S$ , integrating over the neutrino final momenta and summing over the neutrino spins, we easily get the following expression for the energy loss per unit solid angle and per unit volume:

$$Q_{O,H}(\theta) = \frac{1}{12\pi^3} \left( \frac{g^2}{\hbar\alpha} \right) \int d\omega \int_0^{\omega/c} |\mathbf{k}|^2 d|\mathbf{k}| \left[ \frac{\omega^2}{c^2} - |\mathbf{k}|^2 \right] \\ \times (-1)^{\delta\beta 4} \Pi_{\alpha\beta} \Pi_{e\beta}^* e_\alpha e_e^* \left| \frac{\lambda_{ss}^l}{\omega \frac{\partial A^l}{\partial \omega}} \right| f(\omega) \delta(\omega - \omega_\pm), \quad (13)$$

where

$$f(\omega) = \left\{ \exp \left( \frac{\hbar\omega}{KT} \right) - 1 \right\}^{-1},$$

and where we have explicitly introduced the identity

$$\int \delta(x) dx = 1$$

to recall that  $\omega$  is a solution of the dispersion equation, Equation (8).

#### 4. Parallel and Perpendicular Propagation

For  $\theta=0$  and  $\omega=\omega_+=\omega_p$  we have, after some lengthy algebra:

$$Q_O(0) = \frac{1}{384\pi^5} \frac{g^2}{\hbar\alpha} \frac{\omega_p^2}{c^6} f(\omega_p) \int_0^{\omega_p/c} |\mathbf{k}|^2 d|\mathbf{k}| [\omega_p^2 - c^2 |\mathbf{k}|^2]^2 = \bar{Q} \omega_p^9 f(\omega_p), \quad (14)$$

where

$$\bar{Q} = \frac{g^2}{\alpha \hbar c^9} \frac{1}{5040\pi^5}.$$

Integrating over the solid angle, we obtain the formula given by Adams *et al.* (1963), Inman and Ruderman (1964) and Zaidi (1955). For  $\theta=0$  and  $\omega_-=\omega_c$ , we get

$$Q_H(0) = \frac{7}{2} \bar{Q} \omega_p^2 \omega_c^7 f(\omega_c). \quad (15)$$

At this point we would like to comment on the modifications needed when using Equation (15) in the relativistic domain, since the cyclotron frequency becomes velocity dependent. Calling  $\omega_{cR}$  the relativistic cyclotron frequency we have

$$\omega_{cR} = \frac{eH}{mc} \left[ 1 - \frac{v^2}{c^2} \right]^{1/2} \equiv \frac{\omega_c}{\gamma}. \quad (16)$$

Each particle in the plasma now has its own  $\omega_{cR}$  and we have to average over the velocity distribution. For the completely degenerate electron gas we are treating

here, this operation has the effect of replacing  $\omega_c$  in Equation (15) by  $\langle\omega_{cR}\rangle \rightarrow \omega_c/\mu$  where  $\mu$  is the chemical potential of the electron gas in units of  $mc^2$ . Therefore, in the relativistic domain, Equation (15) now reads

$$Q_H(0) = \frac{7}{2} \bar{Q} \omega_p^2 \left( \frac{\omega_c}{\mu} \right)^7 f \left( \frac{\omega_c}{\mu} \right). \quad (17)$$

For  $\theta = \pi/2$  we have essentially one mode,  $\omega = \omega_h$  since the other one,  $\omega = 0$ , gives, of course, a vanishing contribution to the energy loss. Then for  $\theta = \pi/2$ ,  $\omega = \omega_h$  we have

$$Q_o \left( \frac{\pi}{2} \right) = \bar{Q} \omega_p^2 \omega_h^5 \left[ \omega_p^2 + \frac{1}{4} \omega_c^2 \right] f(\omega_h), \quad (18)$$

where  $\omega_c$  has again to be replaced by  $\omega_c/\mu$  in the relativistic domain.

### 5. Propagation at Arbitrary Angles

The general expression for the energy loss Equation (13) can be considerably simplified by expanding it in powers of  $(\omega_c/\omega_p)$ . To the lowest order in  $\omega_c/\omega_p$  Equation (13) becomes

$$Q_H(\theta) = \frac{7}{2} \bar{Q} \omega_p^2 \omega_c^7 f(\omega_-) \cos^7 \theta, \quad (19)$$

where

$$\omega_-^2 = \omega_c^2 \cos^2 \theta.$$

If, furthermore,  $T \gtrsim 10^9$  the energy loss per unit mass becomes

$$Q_H(\theta)/\varrho = 8.24 T (H/H_q)^6 \cos^6 \theta \text{ erg g}^{-1} \text{ sec}^{-1}. \quad (20)$$

In the relativistic domain the preceding formula goes into

$$\frac{1}{\varrho} Q_H(\theta) = 8.24 T \varrho_6^{-2} (H/H_q)^6 \cos^6 \theta \text{ erg g}^{-1} \text{ sec}^{-1}. \quad (21)$$

Equation (21) can be immediately integrated over the solid angle to give for the total luminosity

$$\frac{1}{\varrho} Q_H = 14.8 T \varrho_6^{-2} (H/H_q)^6 \text{ erg g}^{-1} \text{ sec}^{-1}. \quad (22)$$

### 6. Discussion and Numerical Results

The presence of a magnetic field gives rise to two modes of propagation which have called  $\omega_+$  and  $\omega_-$ . The  $\omega_+$  mode is practically independent of  $H$ , and isotropic in the regions of astrophysical interest. In fact all the relevant quantities referring to this mode contain the magnetic field through the combination  $(\omega_c/\omega_p)^2$  which has to be compared to unity.

The  $\omega_-$  mode gives rise to expressions depending strongly on the field and on the angle  $\theta$ . Moreover the density dependence of the energy losses  $Q_H$  and  $Q_o$  is comple-

tely different. In fact, remembering that  $\omega_p \sim \varrho^{1/2}$  and that for high densities  $\mu \sim \varrho^{1/3}$ , we see from Equations (14) and (17) that

$$\frac{1}{\varrho} Q_O \sim \varrho^{7/2} e^{-aT^{-1} \varrho^{1/2}}, \quad (23)$$

while

$$\frac{1}{\varrho} Q_H \sim \varrho^{-2}. \quad (24)$$

Therefore, the energy loss in the presence of a magnetic field can be many orders of magnitude greater than the corresponding loss without field for sufficiently high densities. The energy loss per unit mass at  $\theta=0$  has been computed from Equations (14) and (17), dividing them by the density, for  $H/Hq=10^{-2}$  and  $H/Hq=1$  and several temperatures ( $Hq=(m^2 c^3 / e \hbar) = 4.414 \times 10^{13}$  G). The results have been plotted in Figures 1 and 2 as a function of  $\varrho_6 = 10^{-6} \varrho / \mu_e$ ,  $\mu_e = Z/A$ . In Figure 3 we have reported the total energy losses as deduced from Equations (14) and (22) for  $T=10^9$  and  $H=10^{-2} Hq$ .

As already pointed out whatever sizeable effect the magnetic field is responsible for, it always introduces some degree of anisotropy. In this case, the two factors concur,

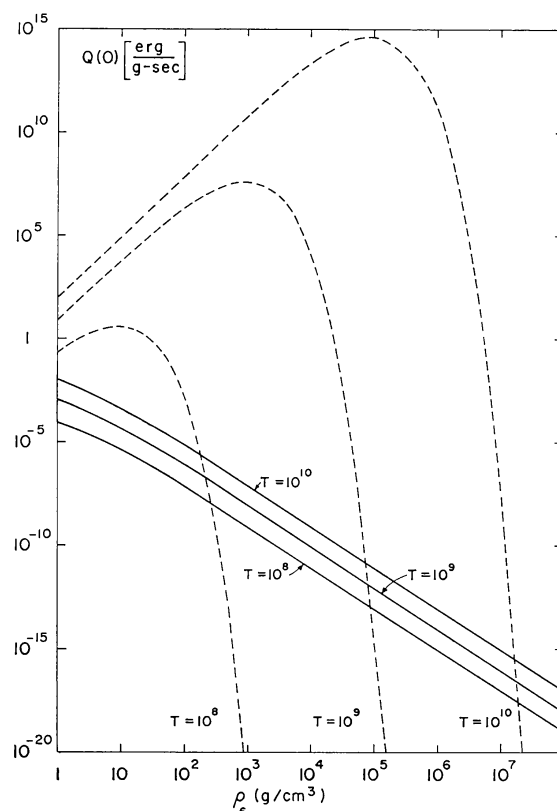


Fig. 1. Energy loss per unit mass and unit solid angle at  $\theta=0$ , as a function of  $\varrho_6$  and different temperatures. The solid curve refers to the mode  $\omega = \omega_e$ , and  $H/Hq = 10^{-2}$  ( $Hq = 4.414 \times 10^{13}$  G). The dashed curve refers to the mode  $\omega = \omega_p$ , which is independent of the field.

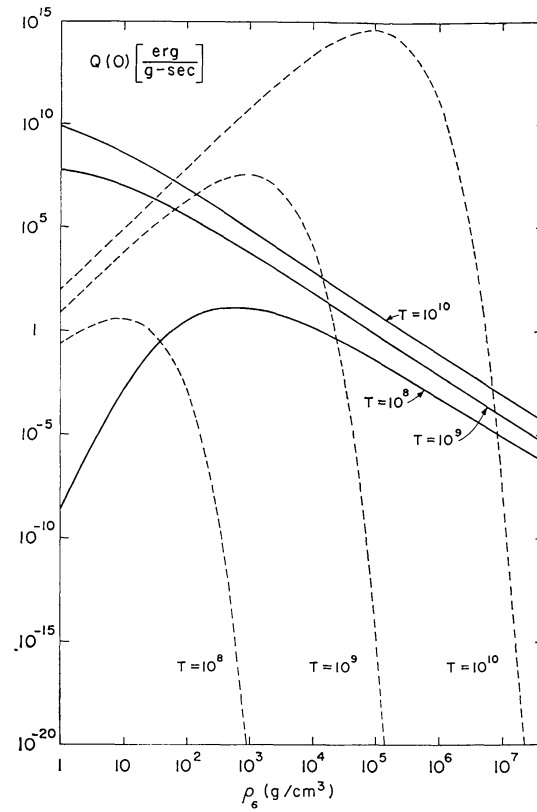


Fig. 2. Same as in Figure 1 for  $H = Hq$ .

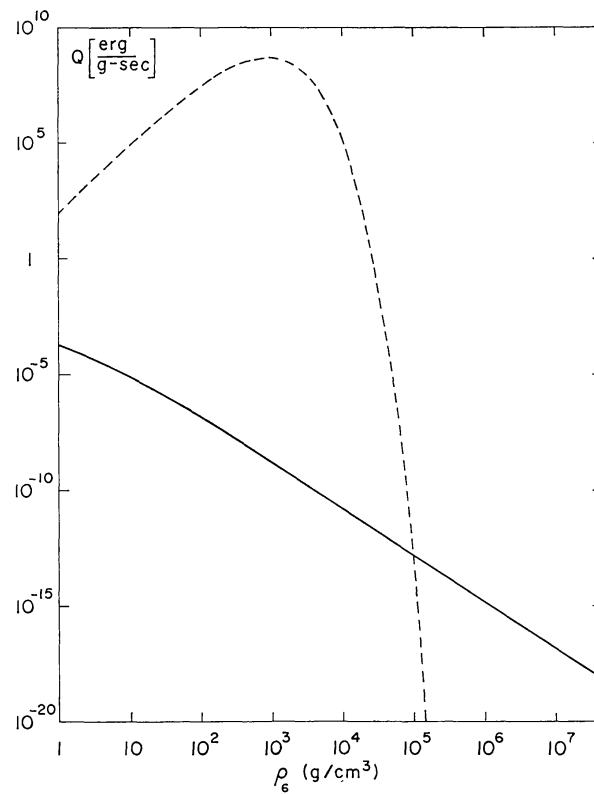


Fig. 3. Total energy losses with (solid line) and without magnetic field vs  $\rho_6$  for  $T = 10^9$  K and  $H/Hq = 10^{-2}$ .

since the mode  $\omega = \omega_-$  which alone is responsible for the total luminosity at  $\rho \gg 10^{11} \text{ g cm}^{-3}$  is strongly peaked around  $\theta = 0$ . When the remaining neutrinos processes (Urca, photoneutrino synchrotron, and pair creation) are evaluated, a complete picture of the weak interaction process in magnetic neutron stars will then be possible and the important question of the cooling time (Canuto, 1970) by neutrinos could then be reinvestigated.

### Acknowledgements

V. Canuto, NAS-NRC Research Associate; C. Chiuderi, ESRO Fellow, and C. K. Chou, NASA Grant N.G.R. 33-008-012, wish to thank Dr. Robert Jastrow for his hospitality at the Institute for Space Studies. One of the authors (C.C.) would like to thank Dr. D. Melrose for private correspondence on the subject of plasma waves.

### References

- Adams, J. B., Ruderman, M. A., and Woo, C. H.: 1963, *Phys. Rev.* **129**, 1383.  
 Canuto, V., Chiuderi, C., and Chou, C. K.: 1970, *Astrophys. Space Sci.* **7**, 407.  
 Canuto, V., Chiu, H. Y., Chou, C. K., and Fassio-Canuto, L.: 1970, *Phys. Rev.* **2**, 281.  
 Canuto, V.: 1970, *Astrophys. J.* **160**, L153.  
 Canuto, V. and Ventura, J.: 1970, to be published.  
 Inman, C. L. and Ruderman, M. A.: 1964, *Astrophys. J.* **140**, 1025.  
 Melrose, D. B.: 1968, *Astrophys. Space Sci.* **2**, 171.  
 Stix, T. H.: 1962, *The Theory of Plasma Waves*, McGraw-Hill, N.Y.  
 Tsytovich, V. N.: 1961, *JETP* **13**, 1249.  
 Zaidi, N. H.: 1965, *Nuovo Cimento* **40**, 502.